ME 360 Control Systems
Armature Controlled DC Motor Transfer Functions

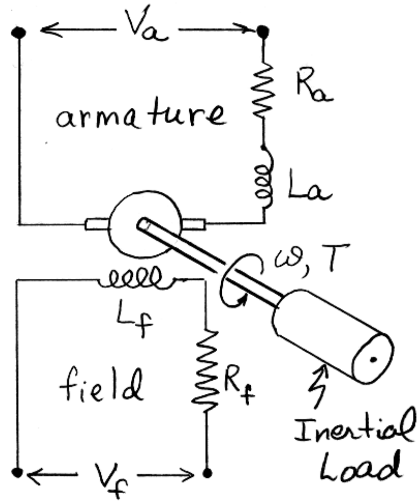
(Reference: Dorf and Bishop, Modern Control Systems, 9th Ed., Prentice-Hall, Inc. 2001)

- In a armature-current controlled DC motor, the field current i_f is held constant, and the armature current is controlled through the ***armature voltage*** V_a.
- The ***motor torque*** increases linearly with the armature current.

$$T_m = K_{ma}\, i_a$$

- K_{ma} is a ***constant*** that depends on a given motor. The ***transfer function*** from the input armature current to the resulting motor torque is

$$\frac{T_m(s)}{I_a(s)} = K_{ma} \qquad (1)$$



DC Motor

- The ***voltage/current relationship*** for the armature side of the motor is

$$V_a = V_R + V_L + V_b = R_a i_a + L_a\left(di_a/dt\right) + V_b \qquad (2)$$

- V_b represents the "***back EMF***" induced by the rotation of the armature windings in a magnetic field. V_b is proportional to the speed ω

$$V_b(s) = K_b\omega(s)$$

- Taking Laplace transforms of Equation (2) gives

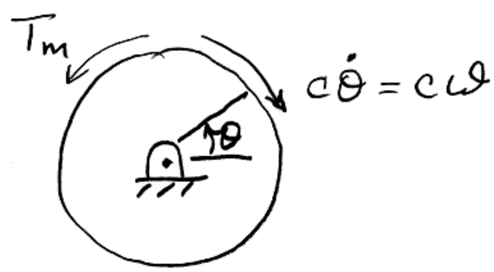
$$V_a(s) - V_b(s) = \left(R_a + L_a s\right)I_a(s) \quad \text{or} \quad V_a(s) - K_b\omega(s) = \left(R_a + L_a s\right)I_a(s) \qquad (3)$$

- An equation describing the ***rotational motion*** of the inertial load is found by summing moments

$$\sum M = T_m - c\omega = J\dot{\omega} \qquad \text{(CCW positive)}$$

or

$$J\dot{\omega} + c\omega = T_m \qquad (4)$$

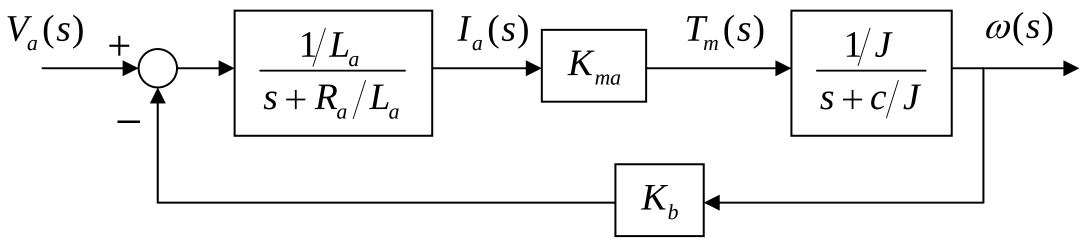


Free Body Diagram
of the Inertial Load

- The transfer function from the input motor torque to rotational speed changes is

$$\frac{\omega(s)}{T_m(s)} = \frac{(1/J)}{s+(c/J)} \qquad \text{(1}^{\text{st}} \text{ order system)} \qquad (5)$$

- Equations (1), (3) and (5) together can be represented by the ***closed loop block diagram*** shown below.



- ***Block diagram reduction*** gives the transfer function from the input armature voltage to the resulting speed change.

$$\frac{\omega(s)}{V_a(s)} = \frac{(K_{ma}/L_aJ)}{(s+R_a/L_a)(s+c/J)+(K_bK_{ma}/L_aJ)} \qquad \text{(2}^{\text{nd}} \text{ order system)} \qquad (6)$$

- If we assume the time constant of the electrical circuit is much smaller than the time constant of the load dynamics, the transfer function of Equation (6) may be reduced to a first order transfer function

$$\frac{\omega}{V_a}(s) = \frac{K_{ma}/R_aJ}{s+(cR_a+K_bK_{ma})/R_aJ} \qquad \text{(1}^{\text{st}} \text{ order system)} \qquad (7)$$

- The transfer function from the input armature voltage to the resulting angular position change is found by multiplying Equations (6) and (7) by $1/s$.